Write your name here


Mathematics AO3
Mathematical problem solving Bronze Test

## Time: 45-60 minutes

Paper Reference
1MA1

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

## Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Answer the questions in the spaces provided
 - there may be more space than you need.
- Calculators must not be used in questions marked with an asterisk (*).
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must show all your working out with your answer clearly identified at the end of your solution.


## Information

- This bronze test is aimed at students targeting grades 6-9.
- This test has 8 questions. The total mark for this paper is 37 .
- The marks for each question are shown in brackets
- use this as a guide as to how much time to spend on each question.


## Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.
*1. Rectangle $A B C D$ is mathematically similar to rectangle $D A E F$.

$A B=10 \mathrm{~cm}$.
$A D=4 \mathrm{~cm}$.
(a) Find the ratio $\frac{A D}{A B}$.
$\qquad$
(b) Use your answer to part (a) to find the distance $A E$.
(c) Thus work out the area of rectangle $D A E F$.
*2. $\quad A(-2,1), B(6,5)$ and $C(4, k)$ are the vertices of a right-angled triangle $A B C$. Angle $A B C$ is the right angle.
(a) Draw a sketch of the triangle described above.
(b) Find an equation for the line $A B$.
(c) Thus find an equation for the line $B C$.
(d) Use your answer to part (c) to find $k$, the $y$-coordinate of $C$.

$$
k=
$$

$\qquad$
(e) Thus find an equation of the line that passes through $A$ and $C$.
(f) Rearrange to give your answer in the form $a y+b x+c$ where $a, b$ and $c$ are integers.
*3. John has an empty box.
He puts some red counters and some blue counters into the box.
The ratio of the number of red counters to the number of blue counters is $1: 4$
Linda takes at random 2 counters from the box.
The probability that she takes 2 red counters is $\frac{6}{155}$.
How many red counters did John put into the box?
(a) Write an algebraic expression for the probability that Linda takes one red counter out of the box.
$\qquad$
(b) Now there is one less counter, write an algebraic expression for the probability that Linda takes a second red counter out of the box.
(c) Use your answers to parts (a) and (b) to create an equation for the probability that Linda takes two red counters out of the box.
$=\frac{6}{155}$
(d) Rearrange and solve your answer to part (c) to find how many red counters John put into the box.
4. A frustum is made by removing a small cone from a large cone as shown in the diagram.


The frustum is made from glass.
(a) Use the formula to work out the volume of the large cone.
$\qquad$ $\mathrm{cm}^{3}$
(b) Work out the height of the small cone.
$\qquad$ .cm
(c) Use similarity to work out the diameter of the base of the small cone.
(d) Thus use the formula to work out the volume of the small cone.
$\mathrm{cm}^{3}$
(e) Use your answers to parts (a) and (d) to work out the volume of the frustum.
$\mathrm{cm}^{3}$

The glass has a density of $2.5 \mathrm{~g} / \mathrm{cm}^{3}$
(f) Work out the mass of the frustum.

Give your answer to an appropriate degree of accuracy.
5. Louis and Robert are investigating the growth in the population of a type of bacteria. They have two flasks A and B.

At the start of day 1 , there are 1000 bacteria in flask A.
The population of bacteria grows exponentially at the rate of $50 \%$ per day.
(a) Write down the population of bacteria at the start of the first few days to show that the population of bacteria in flask A forms a geometric progression.
(b) Write down the common ratio of this geometric progression.

The population of bacteria in flask A at the start of the 10th day is $k$ times the population of bacteria in flask A at the start of the 6th day.
(c) Find an expression for the population of bacteria at the start of the 6th day.
(d) Find an expression for the population of bacteria at the start of the 10th day.
(e) Use your answers to parts (c) and (d) to find the value of $k$.

At the start of day 1 there are 1000 bacteria in flask B.
The population of bacteria in flask B grows exponentially at the rate of $30 \%$ per day.
(f) Sketch a graph to compare the size of the population of bacteria in flask A and in flask B.

(1)
6. In triangle $R P Q$,

$$
\begin{aligned}
& R P=8.7 \mathrm{~cm} \\
& P Q=5.2 \mathrm{~cm} \\
& \text { Angle } P R Q=32^{\circ}
\end{aligned}
$$

Assuming that angle $P Q R$ is an acute angle,
(a) sketch the triangle $R P Q$.
(b) Write down the sine rule.
(c) Substitue values into your answer to part (b) and rearrange to find the angle $P Q R$.

$$
\text { angle } P Q R=
$$

$\qquad$
(d) Hence find the angle $R P Q$.
(e) Write down a formula for the area of a triangle.
(f) Use your answer to part (e) to calculate the area of triangle $R P Q$. Give your answer correct to 3 significant figures.
(g) If you did not know that angle $P Q R$ is an acute angle, what effect would this have on your calculation of the area of triangle $R P Q$ ?
$\qquad$
$\qquad$
$\qquad$
7.

$O M A, O N B$ and $A B C$ are straight lines.
$M$ is the midpoint of $O A$.
$B$ is the midpoint of $A C$.
$\overrightarrow{O A}=6 \mathbf{a} \quad \overrightarrow{O B}=6 \mathbf{b} \quad \overrightarrow{O N}=k \mathbf{b} \quad$ where $k$ is a scalar quantity.
(a) Find $\overrightarrow{O M}$ in terms of $\mathbf{a}$.

$$
\overrightarrow{O M}=
$$

$\qquad$
(b) Find $\overrightarrow{A B}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

$$
\overrightarrow{A B}=
$$

$\qquad$
(c) Find $\overrightarrow{M C}$ in terms of $\mathbf{a}$ and $\mathbf{b}$, giving your answer in its simplest form.

$$
\overrightarrow{M C}=
$$

$\qquad$
(d) Find $\overrightarrow{M N}$ in terms of $\mathbf{a}$ and $\mathbf{b}$.

$$
\overrightarrow{M N}=
$$

(e) Given that $M N C$ is a straight line and a hence multiple of $\overrightarrow{M N}$, find the value of $k$.

$$
k=
$$

8. $\quad V A B C D$ is a solid pyramid.

$A B C D$ is a square of side 20 cm .
The angle between any sloping edge and the plane $A B C D$ is $55^{\circ}$.
(a) On the diagram, find the centre of the base square and mark it $X$.
(b) On the diagram, find the mid-point of $B C$ and mark it $M$.
(c) In the space below, sketch the triangle $A V X$. Mark on it any known angles.
(d) Use Pythagoras to find the distance $A C$.

$$
A C=
$$

(e) Use trigonometry to find the height of the pyramid $V X$.
$V X=$
cm
(f) In the space below, sketch the triangle $V X M$. Mark on it any lengths known.
(g) Use Pythagoras and your answer to part (e) to find the distance $V M$.

$$
V M=
$$

(h) Use your answer to part (g) to find the area of triangle VBC.
$\mathrm{cm}^{2}$
(i) Thus calculate the surface area of the pyramid.

Give your answer correct to 2 significant figures.

## Alternative version of question 3

*3. Hannah has an empty box.
She puts some orange sweets and some yellow sweets into the box.
The ratio of the number of orange sweets to the number of yellow sweets is $1: 4$
She takes at random 2 sweets from the box.
The probability that she takes 2 orange sweets is $\frac{6}{155}$.
How many orange sweets did Hannah put into the box?
(a) Write an algebraic expression for the probability that Linda takes one orange sweet out of the box.
$\qquad$
(b) Now there is one less counter, write an algebraic expression for the probability that Hannah takes a second orange sweet out of the box.
(c) Use your answers to parts (a) and (b) to create an equation for the probability that Hannah takes two orange sweets out of the box.

$$
\begin{equation*}
=\frac{6}{155} \tag{1}
\end{equation*}
$$

(d) Rearrange and solve your answer to part (c) to find how many orange sweets Hannah put into the box.




| Mathematical problem solving: Bronze Test Grades 6-9 |  |  |  |
| :---: | :---: | :---: | :---: |
| Question | Working | Answer | Notes |
|  |  |  | Let $X$ be centre of base, $M$ be midpoint of $A B$ |
| 8 (a-d) | $A C^{2}=20^{2}+20^{2}=800$ | 1300 | P1 process to find $A C$ or $A X$ |
|  | $A X^{2}=10^{2}+10^{2}=200$ |  | P1 process to find $V X$ or $V A$ |
| (e) | $\sqrt{200} \times \tan 55=V X \quad(=20.19 \ldots)$ |  | P1 process to find height of sloping face or angle of sloping face. |
| (f-g) | $V M^{2}=\sqrt{120.19^{12}+10^{2}} \quad(=22.54 \ldots)$ |  | P1 process to find surface area of one triangular face. |
| (h-i) | $4 \times \frac{1}{2} \times " 22.54 " \times 20+20^{2}$ |  | A1 For 1300-1302 |

